DISCRETE PHOTODETECTION AND SUSSKIND-GLOGOWER PHASE OPERATORS

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Abstract

State reduction processes in different types of photodetection experiments are described by using different kinds of ladder operators. A special model of discrete photodetection is developed by the use of superoperators which are based on the Susskind-Glogower raising and lower operators. The possibility to realize experimentally the discrete photodetection scheme in a micromaser is discussed.

1 Continuous and Discrete Photodetections

Usually, photodetection of the single-mode radiation field is described by the use of the mode annihilation and creations operators $\hat{a}$ and $\hat{a}^\dagger$, which can be written in terms of the Susskind and Glogower (SG) [1,2] operators and the number operator $\hat{n}$

$$\hat{a} = \sqrt{\hat{n}+1}\hat{E}_-, \quad \hat{a}^\dagger = \hat{E}_+\sqrt{\hat{n}+1} \tag{1}$$

where

$$\hat{E}_+ = \sum_{n=0}^{\infty} |n+1><n|, \quad \hat{E}_- = \sum_{n=0}^{\infty} |n><n+1|, \quad \hat{E}_-|0>=0 \tag{2}$$

Here $|n>$ are the number states.

When $\hat{a}$ and $\hat{a}^\dagger$ lower or raise a number state $|n>$, they also generate the weight factor $\sqrt{n}$ or $\sqrt{n+1}$, respectively. The SG operators $\hat{E}_+$ and $\hat{E}_-$ only raise or lower the number states without generating any weight factor. The essential difference between the two types of ladder operators implies differences between two photodetection schemes.

In the model of continuous photodetection [3-6] the density matrix of the field is continuously reduced by the information provided by the photodetector. The instantaneous process of one-photon counting is described by the superoperator J:

$$\hat{\rho}(t^+) = J\hat{\rho}(t) \equiv \frac{\hat{a}\hat{\rho}(t)\hat{a}^\dagger}{Tr[\hat{\rho}(t)\hat{a}^\dagger\hat{a}]} . \tag{3}$$

Here $\hat{\rho}(t)$ and $\hat{\rho}(t^+)$ are the density operators for the radiation field immediately before and after the detection. The superoperator J consists of nonunitary transformation (describing state reduction) and the normalization. The no-count process which occurs for a duration time τ is described by the superoperator S_τ:

$$\hat{\rho}(t+\tau) = S_\tau \hat{\rho}(t) \equiv \frac{\exp(-\frac{1}{2}\lambda \hat{a}^\dagger \hat{a} \tau)\hat{\rho}(t)\exp(-\frac{1}{2}\lambda \hat{a}^\dagger \hat{a}\tau)}{\mathrm{Tr}[\hat{\rho}(t)\exp(-\frac{1}{2}\lambda \hat{a}^\dagger \hat{a}\tau)]} \tag{4}$$

Here λ is a parameter characteristic of the coupling between the detector and the field. In continuous photodetection the strength of the interaction depends on the number of photons.

In the present work another photodetection scheme is described in which two-level Rydberg atoms in the lower state are sent through a cavity and their states are measured at the exit. The experimental scheme is similar to that of a micromaser [7]. According to the theories of the micromaser if one starts with a density operator which is diagonal in the number state representation

$$\hat{\rho} = \sum_{n=0}^{\infty} p(n)|n><n| \tag{5}$$

it remains diagonal after the interaction between the radiation and the two-level atoms [8]. We would like to use the information obtained from the measurement of the atoms outside the cavity in order to describe the time development of the field inside the cavity, for a diagonal density matrix. The idea is that in this photodetection scheme the field reduction is described by the superoprator B_- which includes the SG operators

$$\hat{\rho}_{-1} = B_- \hat{\rho} \equiv \frac{\hat{E}_- \hat{\rho} \hat{E}_+}{1 - <0|\hat{\rho}|0>} \tag{6}$$

where $\hat{\rho}$ and $\hat{\rho}_{-1}$ are the density operators for the radiation field before and after the substraction of a photon. The normalization factor is $Tr(\hat{\rho}\hat{E}_+\hat{E}_-) = 1 - <0|\hat{\rho}|0>$. In order to understand why Eq. (6) is valid, we show in the following discussion the differences between the present model of discrete photodetection and the model of continuous photodetection.

In continuous photodetection the measurement occurs continuously at any time whenever the photodetector is active. In discrete photodetection the measurement occurs only when an atom leaves the cavity, so that the number of measurements is equal to the number of atoms transmitted through the cavity. The only referred measurement is that in which an excited atom is detected. Therefore in this model there is no analog to the no-count process of continuous photodetection. In the present model we are not interested in the properties of the interactions inside the cavity and in the associated probabilities. By getting only the information that one atom is excited we reduce an n-photon state of the radiation into an n-1 photon state. The use of Eq. (6) for the density operator of Eq. (5) has only a statistical meaning, where $p(n)$ is the statistical probability that the state is $|n>$ while in fact only one of the states $|n>$ exists in the cavity. For states with different number of photons it will take different times to excite one atom, but by repeating many times the experiments in which one atom is excited and using only the information that one atom is excited the density operator of Eq. (5) is reduced according to Eq. (6). One should take into account that the continuous photodetection theory has also only a statistical meaning. The statistics obtained by that model is exploited by averaging the time development of the system over many quantum trajectories [3]. By getting a different information according to our model we obtain a different photodetection theory which we call discrete photodetection.

As the result of state reduction (6), the changes in the photon number distribution of the radiation field can be expressed in the present model in the following form

$$p_{-1}(n) = < n|\hat{\rho}_{-1}|n > = \frac{< n|\hat{E}_{-}\hat{\rho}\hat{E}_{+}|n >}{1- < 0|\hat{\rho}|0 >} = \frac{p(n+1)}{1-p(0)} . \tag{7}$$

For comparison, the continuous photodetection model gives for the one count process

$$p(n, t^{+}) = < n|\hat{\rho}(t^{+})|n > = \frac{< n|\hat{a}\hat{\rho}(t)\hat{a}^{\dagger}|n >}{< \hat{n} >_t} = \frac{n+1}{< \hat{n} >_t} p(n+1, t) . \tag{8}$$

The mean photon number immediately after the measurement of an excited atom is given according to Eq. (6) by:

$$< \hat{n} >_{-1} = \frac{< \hat{n} >}{1-p(0)} - 1 , \tag{9}$$

while in the continuous photodetection theory the mean photon number immediately after the one count process [6]:

$$< \hat{n} >_{t^{+}} = < n >_t - 1 + \frac{(\Delta n)_t^2}{< \hat{n}_t >} \tag{10}$$

The difference between the continuous photodetection theory and the model of discrete photodetection can be explained also as the difference between a statistical model of matter-radiation interaction by a detector and a statistical model of nondemolition [9] experiments, respectively. The measurements of atoms excitations outside the cavity in the discrete photodetection model gives information only on the change in the number of photons inside the cavity but does not give information on phase changes of the field. This quantum feature follows from the fundamental principle that it is not possible to produce cloning of all the quantum information. Therefore in the present experimental scheme of the micromaser one can get enough information only for diagonal density matrix in which the information on phases has been eliminated [8].

2 Experimental Realization of Discrete Photodetection

We can generalize our model by sending atoms in the lower state through the cavity till the measurement shows a desired number N of excited atoms. Then the field state is reduced according to

$$\hat{\rho}_{-N} = B_{-}^{N}\hat{\rho} \equiv \frac{\hat{E}_{-}^{N}\hat{\rho}\hat{E}_{+}^{N}}{\mathrm{Tr}(\hat{\rho}\hat{E}_{+}^{N}\hat{E}_{-}^{N})} . \tag{11}$$

Our experimental scheme also enables us to add photons to the cavity where in this case we send atoms in the upper state through the cavity and measure their states in the exit till the measurement shows a desired number N of de-excited atoms. Then the field state is reduced according to

$$\hat{\rho}_{+N} = B_+^N \hat{\rho} \equiv \hat{E}_+^N \hat{\rho} \hat{E}_-^N \ . \tag{12}$$

In any real experiment we cannot ignore losses, and the detector of the atoms is not perfect. For imperfect detection we can generalize our model by assuming that the measurement reduces the density operator in the form

$$\hat{\rho}_{\pm\bar{N}} = \sum_N \alpha_N B_\pm^N \hat{\rho} \ . \tag{13}$$

The detector efficiency distribution α_N must be sufficiently narrow around the true number $\bar{N}$ of excited (or de-excited) atoms in order to realize our mode. The validity of the present model of discrete photodetection theory can be checked by doing the experiments with the micromaser in a very special way. Two-level Rydberg atoms which are in the lower state are transmitted through a cavity which is initially in the vacuum state. In many experiments atoms excitations are measured outside the cavity where each experiment is divided into two stages. In the first stage we wait a time t_1 till a fixed number n of atoms is excited. This time is variable from one experiment to another according to quantum mechanical statistical features [8]. However, in each experiment we rescale the time t_1 to a zero initial time and measure in the second stage the number of atoms excitations during an additional time t which is fixed to be the same, for all the experiments in the second stage. Now, we check the prediction of the usual quantum mechanical statistical theory of the micromaser [8] for a time of interaction t, assuming an initial number state $|n>$. The interesting point here is that we can verify by examining the results of the measurements in the second stage that our initial state obtained from the first stage was the number state $|n>$. Such experiments can be done only if the losses are quite small which means that the criterion of a narrow parameter α_N in Eq. (1) is valid.

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